The True Resolution to Russell's Paradox

Formal Outline: Set Theory Remains Unaltered

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1. Abstract

Russell's Paradox is a seemingly impossible situation that occurs in set theory focusing on set A, the set of all sets that do not belong to themselves:

$A = \{a \mid a \notin a\}$

The paradox: set A cannot belong to itself, but it cannot appear to not belong to itself either.

Impossible statement:

$A \in A \Leftrightarrow A \notin A$

Impossible statement:

$A \notin A \Leftrightarrow A \in A$

2. Resolution: The Implicit Membership Identity

One fact is overlooked in the abstract: every element of a set belongs to that set. All sets carry the *implicit membership identity* for each of their elements. The implicit membership identity is never mentioned because it is trivial information already understood to be true. Overlooking the implicit membership identity typically carries no consequences when dealing with sets; dealing with set A is an exception. It must not be taken for granted that every element of set A belongs to set A:

$A = \{a \mid a \notin a\}$

...Application of the implicit membership identity:

$= \{a \mid a \notin a \text{ and } a \in A\}$

Impossible statement:

$A \in A \Leftrightarrow A \notin A \text{ and } A \in A$

Possible statement:

$$A \notin A \Leftrightarrow \text{not}(A \notin A \text{ and } A \in A)$$
$$\Leftrightarrow \text{not}(A \notin A) \text{ or } \text{not}(A \in A)$$
$$\Leftrightarrow A \in A \text{ or } A \notin A$$

3. Conclusion

It is not possible for set A to belong to itself, but it is possible for set A not to belong to itself. The result: "the set of all sets that do not belong to themselves" does not belong to itself. Russell's Paradox only appears to be a paradox when the implicit membership identity is overlooked.